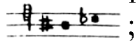
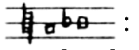
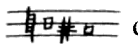
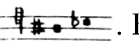


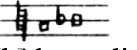
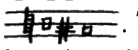
Secunda conclusio: Ipsum comma per se est in clavibus possibile designari, ut ecce ; distat namque prima a c-sol-fa-ut per duo minora, ut in proxima conclusione fuit demonstratum, secunda vero per tonum; sed tonus excedit duo minora per solum comma, ut ex dictis in secunda parte huius opusculi clare patuit, manet igitur inter ipsas solum comma. Potest ergo hec operatio commatis sub hac regula signari: quotiens tonus restrictus est per utramque de b-fa-b-mi, vel minus semitonium per unam, nascitur ipsum comma.

De tono patuit in exemplo predicto, quoniam inter a-la-mi-re et b-fa-b-mi sine litteris est tonus, qui ex utraque parte diminutus est, puta per duo maiora. De semitonio minore ut hic : inter quas est solum comma, eo quod prima distat ab a-la-mi-re tono integro, alia duobus minoribus. Eodem modo hic  quarum prima distat a d-la-sol-re integro tono, secunda vero commate minus. Ex quibus manifestum est, quod ipsum comma in diversis clavibus potest optime signari. Neutrum tamen tetracordum Boetii recipit ipsum comma, de quo satis miror.

Tertia conclusio: In omni commate altioris clavis gravius et bassioris acutius sunt note pronuntiande, ut in omnibus patuit exemplis predictis, in quibus omnibus nota altioris clavis gravius est proferenda quam nota clavis bassioris. Sed quia conclusio supponit, quod impossibile est sic comma disponi, ut nota altioris clavis acutius proferenda sit, idcirco hoc probatur per inductionem:

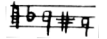
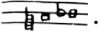
Cum enim per tertiam suppositionem quolibet due claves proxime secundum regularem dispositionem distant tono vel semitonio minore, oportebit necessario, quod in designatione commatis note extra naturas clavium suarum restringantur. Sed si minus semitonium restringitur per

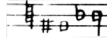
Second conclusion. It is possible for the comma in itself to be designated in the keys, as you can see here: . For the first [note] [a#] has a distance of two minor [semitones] from c-sol-fa-ut, as demonstrated in the previous conclusion, but the second [b] [a distance] of a tone. Now the tone is only a comma more than two minor [semitones], as has been clearly apparent from the things said in the second part of this little work. So between [a# and b] there is only a comma. Therefore the working of the comma can be indicated according to this rule: whenever a tone is diminished by both [of the letters b or #] of b-fa-b-mi, or the minor semitone by one [of those letters], then the comma is born.

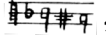
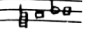
In the case of the tone this has been apparent from the aforesaid example, for without the letters [b or #] there is a tone between a-la-mi-re and b-fa-b-mi, which is diminished on both sides, that is by two major [semitones]. But in the case of the minor semitone like so:  Between these [notes] there is only a comma, for the first [b] has a distance of a whole tone from a-la-re-mi, and the other [c] [a distance] of two minor [semitones]. In the same manner here: . The first of these [c] has a distance of a whole tone from d-la-sol-re, but the second [b#] a comma less. From these things it is apparent that the comma can be indicated very well in various keys. However, neither of Boethius's tetrachords contains the comma, at which I marvel not a little.

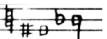
Third conclusion. In every comma, the notes of the higher key are to be performed lower, and of the lower key higher, as has been apparent from all the aforesaid examples. For in all of these [examples] the note of the higher key is to be performed lower than the note of the lower key. Yet since the conclusion presupposes that it is impossible to dispose the comma in such a way that the note of the higher key would be performed higher, therefore this must be shown through induction.

Now since any two neighboring keys, according to the third supposition, have a [mutual] distance of either a tone or minor semitone, [at least] in the regular disposition, [therefore] it shall be necessary to diminish the notes beyond the nature of their keys in order to indicate the comma. Yet if the minor semitone is diminished by one [of the letters] of

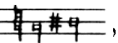
unam de b-fa-b-mi, sequitur per secundam conclusionem, quod nota bassioris clavis acutius et altioris gravius per unum comma decantentur. Si per utramque, tunc per duo commata et unum semitonium minus, ut hic , ubi prima distat ab a-la-mi-re per duo minora per conclusionem primam, secunda vero per tonum cum semitono maiore, nam nisi b-mi littera secundam acueret per maius semitonium supra tonum, quem cum a-la-mi-re sine ipsa littera faceret, frustra b-mi littera ibidem poneretur, ut in prohemio huius tertie partis declaravi; sed tonus cum semitono maiore excedunt duo semitonia minora in duobus commatibus et uno minore. Sequitur igitur necessario, qualitercumque semitonium minus restringitur, quod nota clavis altioris bassius et bassioris altius debeant proferri. Si autem tonus restringitur per unam, veniet minus semitonium, ut hic .

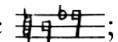
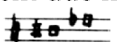
Si per duas, sequetur propositum, quod altioris clavis nota gravius et bassioris acutius necessario proferetur per regulam prime conclusionis. Si autem distantia semiditonalis, qualis est inter c-sol-fa-ut et a-la-mi-re, restringitur per unam, fiet per primam conclusionem duo semitonia minora. Si per duas, deficiet comma a semitono minore, ut hic ; nam sine littera b-fa essent inter eas duo minora, ut iam est dictum, a qua distantia resecat littera b-fa semitonium maius per primam suppositionem; igitur pro semitono minore complendo deficit unum comma. Sed quia semitonium minus plus continet quam tria commata, ut supra in secunda parte huius opusculi est ostensum, manifeste concluditur, quod plus quam comma remanserit inter ipsas. Multo fortius concluderetur, quod plus quam comma debeat remanere, qualitercumque maior distantia quam semiditonalis ut dytonus vel dyatesson restringetur. Patet igitur demonstrata possibilitas signationis ipsius commatis et impossibilitas eiusdem aliter designandi. Utinam ipsum ita infallibiliter perciperem in cantando!

b-fa-b-mi [that is, # or ♭], then it follows from the second conclusion that the note of the lower key would be sung higher, and that of the higher key lower, by one comma. If by both [letters], then by two commas and one minor semitone, as here , where the first [c] has a distance of two minor [semitones] from a-la-re-mi according to the first conclusion, but the second [b#] [a distance] of a tone plus major semitone. For unless the letter b-mi [#] raised the second [note] by a major semitone above the tone which [that note] would make with a-la-re-mi without a letter, the letter b-mi [#] would be placed there for no reason, as I have stated in the prologue to this third part. But the tone plus major semitone exceed two minor semitones by [a distance of] two commas plus one minor [semitone]. Therefore it necessarily follows that no matter how the minor semitone is diminished, the note of the higher key must be performed lower, and [that] of the lower key higher. But if the tone is diminished by one, there will be a minor semitone, .

If by two, then [our] proposition would follow, [namely,] that the note of the higher key would necessarily be performed lower, and [that] of the lower [key] higher, according to the rule of the first conclusion. Yet if the distance of the minor third between c-sol-fa-ut and a-la-mi-re is diminished by one, then according to the first conclusion there would be two minor semitones. If by two, then the minor semitone will lack a comma, as here . For without the letter b-fa [that is, ♭ placed on c] there would be two minor [semitones] between [a# and c], as has been said already, from which distance the letter b-fa [♭] cuts a major semitone according to the first supposition. There is, therefore, one comma lacking to fill the minor semitone. But since the minor semitone contains more than three commas, as has been shown above in the second part of this little work, the conclusion is evident that between them there shall have remained more than a comma. And it may be concluded all the more firmly that there should be left more than a comma, no matter how one would diminish [any] distance greater than a minor third or major third or fourth. We have therefore demonstrated the possibility of indicating this comma and the impossibility of indicating it otherwise. If only I could perceive this so faultlessly while singing . . .

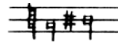
Quarta conclusio: Impossibile est disponi maius semitonium inter claves diversas. Hec conclusio distinguit figurationem cantus cromatici nostram ab antiquis, cum enim nos apothomen, quam ipsi sub duabus cordis statuerunt, in una eademque clave cogimur figurare. Probatur:

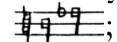
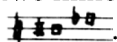
Clarum est, si semitonium minus dilatemus per unam de b-fa-b-mi, quod usque ad tonum ascendemus, ut hic , ubi plus quam semitonium maius existit; multo fortius plus fieret, si maiorem distantiam dilatemus.

Quod autem restringendo quascumque notas diversarum clavium nequimus ad semitonium maius prescise pervenire, <patet> per exempla tertie conclusionis usque ad distantiam semiditonalem; sed per tertiam suppositionem nulle claves reperiuntur distantes prescise tono et duobus semitoniis minoribus. Si ergo distantia ditonalis restringatur per unam litteram, manet tonus cum semitono minore, ut hic ; si vero per utramque, remanent duo minora et sic plus quam semitonium maius, ut hic . Multo fortius probatur conclusio predicta, si maior distantia quam ditonalis, puta dyatesseron, restringenda capiatur.

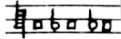
Corollarium, quod ex hac conclusione potest elici, est tale: Impossibile est duo maiora semitonia supra se invicem collocare. Nam cum semitonium maius signabile est, ut in principio huius tertie partis est ostensum, sequitur a contrario sensu eorum, que dicta sunt supra proximo, quod necessario habet fieri inter notas unius et eiusdem clavis; et cum necessario habet fieri cum altera litterarum de b-fa-b-mi per primam suppositionem, sequitur, quod in signatione ipsius altera notarum extorquebitur extra naturam suam per maius semitonium prescise, ut ex prima suppositione cum illis, que primam suppositionem precedunt, sufficienter innotescit. Sed cum semitonia talia, puta duo maiora, de quibus corollarium loquitur, deberent inter tres notas signari sicut inter tres numeros due proportiones et duo gradus inter

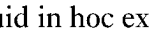
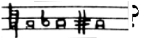
Fourth conclusion. It is impossible to place a major semitone between different keys [rather than within the same key]. This conclusion highlights the difference between our way of indicating chromatic song and [that of] the ancients, for we are constrained to indicate the apothome (which they placed on different strings) in one and the same key. Proof:

It is clear that if we expand the minor semitone through one [of the letters # or b] of b-fa-b-mi, we shall go up to the tone, as here , in which there is more than a major semitone. This would be all the more true if we expanded the greater distance.

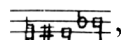
Yet it <is apparent> from the examples of the third conclusion, up to the distance of the minor third, that we cannot arrive precisely at a major semitone by diminishing the notes of different keys. Yet according to the third supposition one cannot find any keys that are distant exactly by a tone plus two minor semitones. So if the distance of the minor third were diminished by one letter, there remains a tone plus minor semitone, as here ; if however by both, then there remain two minor [semitones], and thus more than the major semitone, as here . The aforesaid conclusion is proved all the more firmly when a distance greater than the major third, say, a fourth, is taken to be diminished.

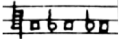
The corollary that can be drawn from this conclusion is as follows. It is impossible to place two major semitones on top of one another. For since the major semitone can be indicated, as has been shown at the beginning of this third part, it follows from the contrary meanings of the things we have said above in the preceding [part], that [the major semitone] must necessarily exist between notes of one and the same key [for example, between b and b $\flat$ , or c and c $\sharp$ ]. And since, according to the first supposition, it must necessarily be made with either of the letters [b or #] of b-fa-b-mi, it follows that its indication causes either of the notes to be turned outside of its nature by [the distance of] exactly a major semitone, as was made sufficiently clear in the first supposition, along with the things that precede the first supposition. Yet since the semitones of which our corollary speaks, that is, two major ones, are to be indicated between three notes, just as two proportions [reside] between three numbers, and

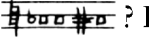
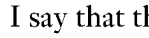
tres personas, quere de illa tertia nota, utrum stet in sua propria natura vel non. Et qualitercumque dicatur ipsa, proportionabitur alteri precedentium in equalitate, ut hic , ubi media et finalis sunt totaliter equales, cum illa secunda littera b-fa non potest plus notam sequentem extra naturam suam extorquere quam per semitonium maius per primam suppositionem; et propria natura eius, quam antiqui sibi attribuerunt, est, quod faciat tonum ultra a-la-mi-re; frustra igitur ponitur secunda littera b-fa.

Sed quid in hoc exemplo: ? Dico, quod prime due sunt equales et tertia ipsas superat in semitonio maiore. Sed quid hic: ? Dico, quod prima et ultima sunt equales, distantes scilicet minore semitonio a clave c-sol-fa-ut, et vincunt mediam semitonio maiore, quia distat ab eadem clave tono integro. Relinquo ergo sufficienti dictione corollarium demonstratum.

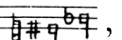
Quinta conclusio: Impossibile est minus semitonium in una clave situari. Nam cum inter quascumque duas notas eiusdem clavis est proportio equalitatis, que equalitas ad inequalitatem torqueri non potest nisi virtute litterarum b-fa vel b-mi et ille demonstrantur in maioribus semitoniis prescise per primam suppositionem, sequitur ergo propositum.

Sexta conclusio: Impossibile est unum tonum per se inter plures claves quam duas residere. Probatur: nam si semiditonalis distantia restringitur per unam, fiet duo minora per primam conclusionem et sic plus; si per duas, deficiet comma a semitonio minore per probationem tertie conclusionis, et sic minus. Sed ditonalis distantia semel restricta causabit tonum cum semitonio minore, quod non est opus declarare; si autem bis restringitur, deficiet a tono unum comma, et sic minus, ut hic , ubi duo semitonia maiora auferuntur a duobus tonis; et cum duo toni continent duo minora et duo maiora, resecatis duobus maioribus remanent duo

two steps [or: degrees] between three persons, you must ask about that third note if it stands in its proper nature or not. And however it may be said, it shall be proportioned with equality to either of the preceding ones, as here , where the middle and final [notes] are completely equal, since the second letter b-fa [b] cannot turn the subsequent note [b] any further outside of its nature than a major semitone, following the first supposition. And the proper nature of [the key b], as ascribed to it by the ancients, is that it should make a tone above a-la-re-mi. The second letter b-fa [b] is therefore placed for no good reason.

But what about this example: ? I say that the first two [b and b] are equal, and that the third [b] exceeds them by a major semitone. But what in this case: ? I say that the first and last [b and b] are equal, having namely the distance of a minor semitone from the key c-sol-fa-ut, and that they exceed the middle one [b] by a major semitone, because [the latter note] has a distance of a whole tone from the same key. I therefore leave the corollary as demonstrated by sufficient discussion.

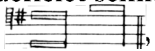
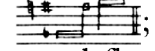
Fifth conclusion. It is impossible for the minor semitone to be situated within one [and the same] key. For there is a proportion of equality between any two notes in the same key, and this equality cannot be turned into inequality except by virtue of the letters b-fa [b] or b-mi [#]. And these [letters] are shown in the first presupposition to [effect] exactly a major semitone. Therefore the proposition follows.

Sixth conclusion. It is impossible for one tone by itself to reside between more keys than two. Proof: if the distance of a minor third is diminished by one [letter b or #], then according to the first conclusion there shall be two minor [semitones left], and thus more [than a tone]. If by two [letters], then according to the proof of the third conclusion the [remaining] minor semitone will be one comma short, and thus less [than a tone]. On the other hand, if the distance of a major third is diminished once, it shall produce a tone plus minor semitone, which it is not necessary to explain. If however it is diminished twice, the [remaining] tone shall be one comma short, and thus less, as here , where two major semitones are taken away from two tones. And since two tones contain [altogether] two minor and two major [semitones], there remain two minor ones after the

minora; sed duo minora solo commate deficiunt a tono, ut in secunda parte huius ostensum est. Relinquitur ergo conclusio demonstrata.

Septima conclusio et ultima quoad hanc materiam sit ista: Impossibile est dyapason inter plures paucioresve claves statuere quam octo. Probat: Quelibet enim octava clavis regulariter ad aliam se habet in proportione dupla, ut patet discurrendo per omnes viginti, et ob hoc una eademque littera communiter designantur ut G-ut et g-sol-re-ut, A-re et a-la-mi-re et sic de singulis.

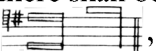
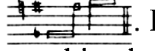
Sint ergo I et L due note disposite inter octo claves, que faciant dyapason precise, et sit K nota distans ab I in minori distantia, id est inter pauciores claves, puta in clave septima, tunc per tertiam suppositionem oportebit, quod inter K et L sit tonus vel semitonium minus, et ergo per tantumdem minor est distantia inter K et I quam sit inter I et L.

Si vero inter K et L sit minus semitonium, necessarium erit distantiam inter K et I dilatari, ut veniat ad equalitatem cum distantia, que est inter L et I. Sed si semel dilatetur, id est per unam litteram de b-fa-b-mi, que per primam suppositionem addit semitonium maius precise, sequitur, quod distantia inter K et I vincet dyapason consonantiam in uno commate per ultimum exemplum conclusionis secunde; multo fortius, si predicta distantia per utramque dilataretur. Sed pone, quod inter K et L sit tonus. Non poterit distantia inter K et I per unam dilatari, quoniam deficiet semitonium minus a proportione dupla, ut hic , ubi media distat ab ultima in semitonio minore, ut vidimus in ultimo exemplo conclusionis quarte; deficit igitur tantumdem a proportione dupla, que est inter primam et ultimam. Si vero per duas dilatetur, excedet ipsam dyapason in solo commate, ut hic , nam si littera b-fa non esset apposita, ipsi prime non deficeret nisi minus semitonium, ut constitueret dyapason cum secunda

two major ones are cut away. But two minor ones are only a comma less than the tone, as shown in the second part of this [treatise]. Which leaves the conclusion demonstrated.

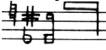
The seventh and last conclusion concerning this matter is the following. It is impossible to place an octave between more or fewer keys than eight. Proof: it is the rule that any eighth key relates to the other [first key] in duple proportion, as is apparent when one runs through all twenty [keys]. And because of this they are commonly designated by one and the same letter, like G-ut and g-sol-re-ut, A-re and a-la-mi-re, and thus the others.

Let I and L therefore be two notes arranged between eight keys which span exactly an octave. And let K be a note at a smaller distance from I, that is, between fewer keys, for example, on the seventh key. Then it shall be necessary, following the third supposition, that between K and L there is a tone or minor semitone. And therefore the distance between K and I is less than that between I and L.

But if there is a minor semitone between K and L, it shall be necessary for the distance between K and I to be widened in order to make it equal to the distance between L and I. But if it were to be widened once, that is, by one letter of b-fa-b-mi (which, according to the first supposition, adds exactly a major semitone), it follows that the distance between K and I shall exceed the consonance of the octave by one comma, following the last example of the second conclusion. Much more so, then, if the aforesaid distance would be widened by both [letters]. Now suppose that between K and L there is a tone. It shall not be possible for the distance between K and I to be widened by one, for there shall be a minor semitone less than duple proportion, as here , where the middle [note] is a minor semitone away from the last [note], as we have seen in the last example of the fourth conclusion. Therefore there is just as much lacking from the duple proportion between the first and last [notes]. But if it were to be widened by two, this very octave has an excess of one comma, as here . For if the letter b-fa [b] had not been placed, there would be nothing lacking from the first [note] except a minor semitone, to make up an octave with

nota, ut modo dicebam; sed b-fa littera nimis large addit, quia semitonium maius, quod excedit ipsum minus semitonium in commate supradicto.

Pone tamen, quod K et I distent a seinvicem in sexta clave, tunc per tertiam suppositionem inter K et L duo erunt toni vel tonus cum semitono minore. Si duo toni, illi addi non possunt etiam per utramque litteram de b-fa-b-mi, quoniam deficient duo minora, ut vidimus in exemplo conclusionis sexte.

Si tonus cum semitono minore, deficient duo commata et amplius, non tamen tria, etiam si per utramque litteram distantia inter I et K dilataretur, ut hic ; est enim semiditonalis distantia maior duobus semitoniis maioribus in duobus commatibus et amplius, non tamen in tribus, ex quo semitonium minus continet plus quam tria commata, non tamen quatuor perfecte, ut ex secunda parte huius opusculi clare patet intuenti; que duo commata, etiam si quatuor essent, non maiori semitono possunt adequari.

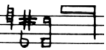
Relinquitur igitur altera pars conclusionis probata, videlicet quod dyapason sub paucioribus clavibus quam octo nullo modo possibile sit disponi. E converso totaliter demonstrabile est ad oculum, quod nec inter plures claves quam octo dyapason poterit ordinari: a cuius probatione propter prolixitatem vitandam ad presens supersedeo, cum probatio huiusmodi satis liqueat ex predictis.

Patet ergo, quod propriissime et non sine ratione vocamus nos Teutonici dyapason octavam. Pari ratione dyapente quintam et dyatesseron vocamus quartam a numero clavium, inter quas precise necessarium est ipsas poni. Sic et tertiam tertiam et sextam non alio nomine vocamus quam sextam, de quibus omnibus nove possunt fieri conclusiones, que de se, visis hiis, que facte sunt, satis liquide relucescunt.

Apud tamen chromaticos, qui tonum inter tres cordas et triemitonium inter duas statuunt, item Enarmonicos, qui

the second note, as I just said; yet the letter b-fa [b] adds too much to the large one, namely, a major semitone, which exceeds the minor semitone by the abovesaid comma.

Suppose, however, that K and I had a distance from each other [defined by] the sixth key. Then following the third supposition there shall be two tones, or a tone plus minor semitone, between K and L. If two tones, then these cannot be added even by both letters of b-fa-b-mi, as they shall be two minor ones short, as we have seen in the example of the sixth conclusion.

But if [there is] a tone with minor semitone [between K and L], then they lack two commas and a little more, but not three, even if the distance between I and K were to be widened by both letters, as here ; for the distance of the minor third is greater than [the distance of] two major semitones, by [a difference of] two commas and a little more, but not three. From this [it appears that] the minor semitone contains less than three commas, but not four perfect ones, as is clearly apparent to the observer from the second part of this little book. Which two commas, even if there were four [of them], cannot be made equal to a major semitone.

The second part of the conclusion, namely, that the octave cannot in any way be disposed between fewer keys than eight, is therefore left as having been proved. Conversely it is totally demonstrable as obvious that it shall neither be possible to arrange the octave between more keys than eight. I pass over its proof presently, to avoid prolixity, since this proof flows amply from the aforesaid things.

It is apparent, therefore, that it is most proper, and not without reason, that we Germans call the diapason an octave. For the same reason we call the diapente a fifth, and the diatessaron a fourth, after the number of keys between which it is necessary to place them precisely. Thus we also call the third a third and the sixth a sixth, and not by any other names. From all of these things there are new conclusions to be drawn, which are sufficiently obvious, given the things we have [already] done.

However, among the chromatics, which place the tone between three strings and the trihemitone between two, and also the enharmonics, which

lymmati tres cordas et ditono duas tribuunt, non est dictis nominibus taliter utendum: vocant ergo triemitonium et ditonum utrumque incompositum propter carentiam tertie corde. Possunt etiam inferri ex hac conclusione corollaria: quod aliquotiens maior est distantia sex clavium quam octo et aliquotiens maior octo quam decem, et multa similia, que studiosis duxi totaliter relinquenda.

Antequam tamen finem presenti materie imponam, ut remorsus me stimulans circa suppositionem secundam apud omnes sciatur, adicere volo pauca. Habet enim secunda suppositio, quod littere ipsius **b-fa-b-mi** undique situari possunt ad placitum cantum conficientis, quo supposito sunt conclusiones supradicte cum earum probationibus omnes clare.

Sed pone, quod non undique hoc liceat, sicut bene capit ymaginatio et persuadet ratio, quod non ante notam, que cum proxima facit semitonium minus, utputa quod non liceat poni **b-mi** litteram in clave **b-mi** nec in **e-la-mi** aut in octavis, id est duplis, ad istas, nisi in casu, quo **b-fa** littera in eisdem clavibus precessisset, quod tunc liceret, ut effectus littere **b-fa** precedentis per litteram **b-mi** extingueretur. Ratione suadente, quia **c-fa-ut** et **f-fa-ut** cum duplis suis faciunt cum dictis clavibus semitonium minus; quareque non liceat poni **b-fa** litteram in **c-fa-ut** vel in **f-fa-ut** nec in duplis ad istas propter causam iam dictam. Pari ratione nec liceat in **a-la-mi-re** poni litteram **b-mi**, dum **b-fa** littera in clave **b-fa-b-mi** situatur, et sic de similibus.

Concludetur evidenter, quod minor distantia inter notas dari non poterit quam semitonii minoris. Et hec forte conclusio antiquis magis est accepta, qui, dum alte lateque consideraverant gradus strictos et amplos, quibus, et non aliis, natura genus humanum a gravi ascendere precipit in acutum et econtra, experti sunt semitonium minus strictiores comprehendere terminos et idcirco ipsi commati nullas claves

ascribe three strings to the limma and two to the major third, the said names are not to be used in this way. Therefore they call both the trihemitone and the major third non-composite because of the lack of a third string. But there are corollaries that can be inferred from this conclusion. [For example,] that sometimes the distance of six keys is greater than eight, and sometimes [the distance] of eight greater than ten, and many similar things, which I leave to the studios to pursue.

Yet before making an end to the present matter, I wish to add a few things, in order that the remorse which urges me on about the second supposition shall be known by all. For the second supposition holds that the letters of **b-fa-b-mi** can be placed anywhere, at the pleasure of the one who makes the song. And when this is supposed, then the aforesaid conclusions with their proofs are all clear.

But suppose that it would not be permissible everywhere [to place a letter] before the note which makes a minor semitone with the neighboring one, as the imagination easily perceives and reason persuades. For example, that it would not be permissible to place the letter **b-mi** [**#**] on the key **b-mi** nor on **e-la-mi** nor on their octaves (that is, duplas), except if the letter **b-fa** [**b**] had preceded in the same keys, in which case it would be permitted, since the effect of the letter **b-fa** [**b**] would be extinguished by the letter **b-mi** [**#**]. But reason urges, because **c-fa-ut** and **f-fa-ut**, together with their duplas, make a minor semitone with the said keys. And therefore it would not be permissible for the letter **b-fa** [**b**] to be placed on **c-fa-ut** or on **f-fa-ut**, nor on their duplas, because of the reason already mentioned. For the same reason it would not be permissible either to place the letter **b-mi** [**#**] on **a-la-re-mi** while the letter **b-fa** [**b**] is situated on the key **b-fa-b-mi**, and likewise in similar cases.

It may be manifestly concluded that one cannot assign a distance between notes that is smaller than the minor semitone. And this conclusion has perhaps been more fully accepted by the ancients. For when they considered deeply and widely the small and large steps by which (and not others) nature teaches the human genus to ascend from low to high and the other way round, they discovered that the minor semitone has narrower boundaries. And for that reason, since they did

decernentes – quamvis de ipso per accidens, ut dictum est, mentionem faciunt – in semitonio minori tamquam minimo intervallo – sicut in vocibus, ita et in clavibus, ut signum signato respondeat – finaliter quieverunt. Que, inquam, conclusio, si vera sit, destruet nondum suppositionem predictam, verum secundam et tertiam conclusiones cum suis probationibus, ceteris omnibus conclusionibus, quamvis earum probationes, que de commate faciunt mentionem, lese sunt, remanentibus inconcussis.

Sed ne dicta conclusio lateat indiscussa, est subtilius advertendum, quod modernus usus dictas litteras in clavibus extra naturam monocordi manualis admittit solum propter consonantias vel lasciviam ipsius cantus — non enim tantis olim quantis nunc in prolacione practica alicuius cantilene lasciviis hominis inhyarunt — ; et ut hec lasciva iocunditas absque omnimoda novi monocordi compositione signari possit in scriptis, sicut habet fieri in sonis, ut signum signato respondeat, rationabiliter eas litteras et earum effectus in diversis clavibus usus admisit. Nusquam tamen regulariter admittit, quod aliquod de tetracordis ipsam in manu circueuntibus plus quam per apothomen extendatur vel reiiciatur, et hoc, ne nimia mollicies per remissionem vel nimia duricies per extensionem oriatur — remissio autem tetracordi ad tonum cum duobus limatibus apud antiquos nusquam reperta est; sed extensionem eius usque ad tritonum, quam antiqui solum voluerunt in clave b-fa-b-mi, nos in aliis clavibus propter dictam causam et per consequens remissionem tetracordi, quod in eadem clave, qua fit extensio, extenso coniungitur, ministerio dictarum litterarum duximus admittendam — . Et sicut ipsum tetracordum quodcumque, quo dyatesseron residet, nonnisi ea parte, qua mollius est, extendimus ob duriciem vitandam, sic ipsum, ubi durius est, remittimus propter molliciem nimiam fugiendam. Durior pars tetracordi est illa, que ditonum perficit; mollior, que semitonium minus. Ubi ergo indifferens est, id est

not determine any keys for the comma (although they do make mention of it, as said already, in an accidental sense), they finally settled on the minor semitone as the smallest interval – as in pitches, so also in keys, so that the indication should correspond with what is indicated. If this conclusion were to be true, I say, then it shall not quite destroy the aforesaid supposition, nor, in truth, the second and third conclusions with their proofs [and] all their other conclusions, even though the proofs that make mention of the comma are injured, the remaining ones being unaffected.

Yet lest the said conclusion would remain undiscussed, one should note, more subtly, that modern custom admits the said letters in keys outside the nature of the monochord of the hand, but only because of consonances or because of licentiousness of song — for in the past the practices of men in the performance of song did not crave as many licentious things as [they do] now. And in order that this licentious pleasantness may be indicated in writings, without all manner of composition of a new monochord, custom has rationally admitted these letters and their effects in different keys, just as happens with sounds, so that the indication corresponds with what is indicated. However, [modern custom] never regularly allows that something may be stretched or relaxed by more than an apothome from the tetrachords surrounding it in the hand, lest too much softness be created by remission or too much hardness by stretching — for among the ancients one has never found remission of the tetrachord to the tone with two limmata. But we have considered that the extension up to the tritone (which [interval] the ancients only wanted on the key b-fa-b-mi) may also be allowed on other keys because of the said reason, and consequently [also] the remission of the tetrachord, that it is joined to the extended on the same key in which the extension takes place, through the help of the said letters. — And just as we extend any tetrachord in which the fourth resides only in the part where it is softer, in order to avoid hardness, so do we remit it [only] where it is harder, in order to avoid too much softness. The harder part of the tetrachord is that which perfects the major third; the softer [is] that which [perfects] the minor semitone. Where it is indifferent,



equedurum propter semitonium medio loco situatum, per predictam rationem indifferenter, ut apparet, extendi poterit et remitti.

Hec igitur ratio debet sufficere, quare in clave G-ut, que tetracordum, quod cum clave c-fa-ut constituit, ad partem duriolem terminat, non admittit usus poni litteram b-fa sed b-mi, in a-re et d-sol-re nunc unam nunc aliam quamlibet indifferenter, in b-mi et E-la-mi b-fa et non b-mi.

Sed quid in illis tetracordis, que clavem b-fa-b-mi obumbrant? Respondeo pro presenti, quod, quantum tangit hanc materiam, debemus considerare dictam clavem b-fa-b-mi, ut resonet dyapason ad tertiam clavem ipsius manus et per consequens tritonum ad f-fa-ut grave et dyatesseron cum e-la-mi clave acuta. Et ob hoc in g-sol-re-ut et c-sol-fa-ut ponimus solum b-mi sicut in g-ut et c-fa-ut. Et quamvis d-la-sol-re ut d-sol-re utramque indifferenter recipit, tamen a-la-mi-re sola b-fa littera est contenta, ne tetracordum inter a-la-mi-re et e-la-mi extendatur per b durum a parte duriori. Sic ergo non est semper simile in hac materia de clavibus dyatesseron constituentibus vel dyapason, sed solum attenditur mollicies vel duricies tetracordi. Et si queras, quomodo ergo signabitur dupla supra clavem a-re per b-mi elevatum, respondeo consulendo: scimus, quod huiusmodi cantus sub aliis clavibus, quibus hoc licitum est, ordinetur.

Concludo ergo ipsum comma inter aliquas claves, utputa inter G et a, c et d, d et e — licet non in illis omnibus, quas supra gratia exempli solum posui — possibiliter disponendum.

#### <QUARTA PARS>

Consonantia est dissimilium inter se vocum in unum redacta concordia. Vel sic: Consonantia est acuti gravisque soni mixtura suaviter uniformiterque auribus accidens. Utramque

therefore, that is, equally hard because the semitone is placed in the middle location, it may be extended and remitted indifferently, it appears, because of the aforesaid reason.

This must therefore be sufficient reason [to explain] why custom does not allow the letter b-fa [b] to be placed on the key of G-ut (which ends, on the harder part, the tetrachord which it makes with the key c-fa-ut), but only b-mi [♯], [and] on a-re and d-sol-re now one, now the other, indifferently, [and] on b-mi and E-la-mi [only] b-fa [b] but not b-mi [♯].

Yet what about tetrachords that conceal the key of b-fa-b-mi? I respond for now that as far as this matter is concerned, we must consider the said key b-fa-b-mi, as the octave would resound to the third key of the hand, and consequently the tritone to low f-fa-ut, and the fourth to e-la-mi in the high key. And because of this we place only b-mi [♯] on g-sol-re-ut and c-sol-fa-ut, just as on g-ut and c-fa-ut. And although d-la-sol-re like d-sol-re receives both [letters] indifferently, for a-la-re-mi the letter b-fa [b] is sufficient, lest the tetrachord between a-la-re-mi and e-la-mi be extended by hard b on the harder part. In this matter, therefore, it is not always the same with keys that constitute the fourth or octave, but only the softness or hardness of the tetrachord is taken into account. And if you ask how the dupla [proportion] above the key a-re which is raised by b-mi [♯] will therefore be indicated, I respond, counseling: we know that such song would be arranged under different keys, in which this is permissible.

I therefore conclude that it is quite possible for this comma to be disposed among certain keys, as between G and a, c and d, d and e — though not in all the ones which I have posited above by way of example.

#### <FOURTH PART>

Consonance is the concord of dissimilar sounds rendered into one. Or: consonance is the blend of high and low sound falling smoothly and uniformly upon the ears. Boethius, and after him Johannes de

descriptionem predictam ponit ipse Boethius et Johannes de Muris post ipsum. Ex quibus faciliter elicitur unisonum, in quo nulla vocum dissimilitudo aut gravitas vel acumen existit, non consonantiam posse dici. Plato per Nychomacum reprehensus opinabatur tamen unisonum fore causam consonantie, dum dicebat acutum sonum ab hoc cum gravi consonantiam facere, quia non tam celeri impetu in fine sicut in principio brevior corda aerem percutit, sed iam segnior reversa graviori sono similem emittit [Boethius, *Musica*, i. 30, though explained quite differently here]. Nychomachus vero similes son[o]s nullam consonantiam ait facere, sed eam inter sonos dissimiles existere, dumtamen velocitas corde brevioris ceteris paribus se habeat ad velocitatem seu tarditatem corde longioris in proportione convenienti; que autem sit illa proportio, posterius declarabo.

Consonantie ergo apud nos septem sunt genera, scilicet quatuor per se, tria vero per accidens. Genera per se sunt tertia, quinta, sexta et octava, quolibet nomine ad suum fundamentum, puta gravissimam notam, relato, quam primam iuxta modum loquendi nostrum ut Teutonicus appello. Genera per accidens inferius subsequenter.

Et quamvis super octavam consonantes voces ut decimam, duodecimam, tertiamdecimam et bisoctavam seu quadruplam et sic ulterius, que equebene predictas sapiunt descriptiones iudicio aurium, reperimus, non tamen de eis de consonantiarum tractans generibus specialiter faciam mentionem, eo quod sic in consonantiis non esset finis, si ad infinitas vox vel instrumenta possent coaptari. Cui infinitati non est opus insistere, cum inter duos quoslibet sonos in dupla constitutos proportione natura tantam commixtionis aptitudinem disposuit, quod in hac arte presenti unus alter naturali quodam instructu esse supponitur, et nichil ultra octavam, id est duplam proportionem, existit, quin ad aliquid infra eius limites relatum cum eo ydemptitatem quodammodo procreet, ut quidquid de decima, quantum presentem tangit materiam, possumus dicere, hoc idem de

Muris, posits both aforesaid descriptions. From this it is easily inferred that the unison, in which there is no dissimilarity of sound, that is, [no] lowness and sharpness, cannot be called a consonance. Plato (criticized by Nicomachus) was however of the opinion that the unison is the cause of consonance, when he said that a high sound makes a consonance from this [unison] with a low [sound], because a shorter string does not strike the air with so fast an impetus at the end as [it does] at the beginning, but once reversed it emits, more slowly, one similar to lower sound. Nicomachus however says that like sounds make no consonance at all, but that this [consonance] exists [only] between dissimilar sounds, so long as (other things being equal) the velocity of the shorter string relates to the velocity or slowness of the longer string in a correct proportion. I shall later explain which is that proportion.

Amongst us, then, there are seven genera of consonance, namely, four [that are] inherently [consonant], and three [that are consonant] in an accidental sense. The inherently [consonant] genera are the third, fifth, sixth, and octave, each named in relation to its foundation, that is, the lowest note, which I as a German call “the first” after our manner of speaking. The genera in an accidental sense will follow below.

And although we encounter consonant sounds above the octave, such as the tenth, twelfth, thirteenth, and double octave or quadruple, and so forth, which fit the aforesaid definitions equally well according to the judgement of the ears, I shall not make mention of them when dealing more particularly with the genera of consonance, for then there would be no end to consonances, since the voice or instruments can be adapted infinitely. And there is no need to dwell on infinity, for nature has disposed such aptitude of blending, between any two sounds [that are] related in dupla proportion, that in this present art they are understood to be somehow naturally inbuilt in each other. And nothing exists beyond the octave, that is, [beyond] dupla proportion, or [it is] related to something beneath its boundaries, and creates with it some sort of identity. So whatever we can say about the tenth, as far as it touches the present matter, [we can say] this also about the third

tertia, ad quam decima est dupla, quod de duodecima, hoc de quinta, quod de tertiadecima, hoc de sexta, et quod de quadrupla, hoc idem de dupla, et sic sine fine natura dicere nos compellit.

Harum autem quatuor consonantiarum genus dulcius et optimum est octava, quam duplam Gallici, dyapason Greci nomine tracto a proportione denominant propter nimiam similitudinem, ymo quasi ydemptitatem inter suos terminos. Sed unde tanta similitudo provenit, natura novit, et ita possumus persuadere: Ex quo enim corda dupla brevior, celerius ceteris paribus percutit aerem; quantum amittit in longitudine, recuperat in velocitate, et sic facta recompensatione inter sonos ipsarum cordarum maxima similitudo et quasi ydemptitas videtur nasci. Item in divisione alicuius continui facienda pronior est oculus ad eius medietates quam ad tertias; sic est auris pronior ad duplam quam ad sesquialteram. Sic ergo natura voluit, quod ipsa octava, qualitercumque multiplicetur, eadem consonantia m[a]neat et omnes octave unum sonum et eundem quasi creare videantur.

Quinta vero post hec perfectius sonat, prout de hoc testimonium fert auditus. Ab istis duabus consonantiis et in ipsis quilibet cantus regulariter solet incipi atque terminari, quia iste due cantum perficiunt, quapropter eas melicas Boethius vocat; et alie due, puta tertia et sexta, que et portimelice possunt appellari, ipsas quodammodo famulantur.

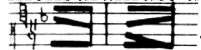
Tertia et sexta imperfectiores quam que predictae sunt consonantias faciunt. Et quia sexta tantum distat ab acuto sono proportionis duple, quantum tertia ab eius sono graviori, equebonas consonantias tertias et sextas ratio satis dictat. Sed quanto minus tertia et sexta in se claudunt dulcedinis, tanto uberius ipsis prospectum est in hoc, quod utraque, tam tertia quam sexta, duplici gaudere possit sonorum ordinatione. potest enim tertia esse dytonalis vel semiditonalis, sic et sexta potest facere tonum vel semitonium minus supra quintam, secundum

of which the tenth is the dupla; and what [we can say] about the twelfth, this also about the fifth; and what about the thirteenth, this about the sixth; and what about the quadrupla, this also about the dupla, and thus nature compels us to speak without end.

The sweeter genus of these four consonances, however, and the best one, is the octave, which the French call dupla, [and] the Greeks diapason, the name having been derived from the proportion because of the great similitude or, rather, virtual identity, between its terms. Yet whence so great a similitude arises, nature knows, and we may argue as follows. The shorter dupla string strikes the air more rapidly, other things being equal, [so] what it lacks in length it recovers in velocity. And thus the greatest similitude, and virtual identity, is seen to be born because a recompense has been made between the sounds of those strings. Also, when some continuum is divided, the eye is more attentive to its halves than to the thirds; in the same way the ear is more attentive to the dupla than to the sesquialtera. Therefore nature wanted this octave to maintain the same [level of] consonance, however [many times] it may be multiplied, and thus all octaves seem to create almost one and the same sound.

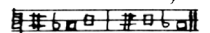
After [the octave], however, the fifth is [also] as more perfect sounding one, according to the testimony of the hearing. Any song is wont to begin regularly with the two consonances and to end in them, since those two perfect the song. For this reason Boethius call them *melica*. And the other two, that is, the third and sixth, which can be called *portimelica*, are servants to them in some way.

The third and sixth make consonances that are more imperfect than those mentioned before. And because the sixth is equally far removed from the high sound of the dupla proportion as the third is from its lower sound, reason decrees that thirds and sixths are equally good consonances. Yet although the third and sixth contain less sweetness within themselves, the outlook is all the more fertile for them in that both, third as well as sixth, may rejoice in a twofold arrangement of sounds. For the third can be either major or minor; and similarly the sixth can make either a tone or a minor semitone above the fifth. And

quod ille proportiones, quibus serviunt, sibi exigunt famulari; nam sicut regularius, dum ad graviolem partem ipsius proportionis duple tendimus, utimur tertia, que minus ab huiusmodi parte distat, puta semiditonalis, sic, dum in acutam partem finire volumus, utimur sexta, que in distantia equali ab ea parte acuta residet, puta semiditonalis, que tonum integrum facit supra quintam; e contra autem sicut dum ad quintam tendimus, distantiam ipsius tertie a graviore dilatamus utendo tertia ditonalis, sic distantiam consimilem, dum ad quintam tendimus, inter octavam et sextam penitus mensuramus, ut de hoc possunt subici exempla sub hac forma 

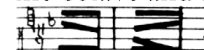
Fratrisesaverunt ergo tertia et sexta in hiis, que dicta sunt, et in hoc, quod quelibet voluit manere sub genere proportionis superpartientis. De dytonali distantia, quam perfectam esse dicimus, supra in b-mi demonstravi. Semiditonalis vero vulgariter eo dicitur, quod constat ex tono et semitono minore. Vocat tamen Boetius etiam magis proprie eam triemitonalem, quia ex tribus semitoniis consistit, duobus scilicet minoribus et uno maiore, quale intervallum seu distantia est inter a et c, id est a-re et c-fa-ut. Et hoc taliter invenitur, si a sesquitertia proportione sesquioctavam resecamus; verbi gratia: proportio 36 ad 27 est sesquitertia. inter quos numeros capiam 32, ad quem numerum se habent 36 in proportione toni, que sesquioctava dicitur; qua proportione reiecta de sesquitertia, que dyatesseron constituit, remanebit proportio, quam queris, puta tertie semiditonalis, inter 32 et 27, que vocatur superpartiens quinque vicesimas septimas.

— Et non sunt plures tertie quam hee due, que dicte sunt, sicut nec alie sexte quam supradicte, quod experientie relinquo auditis. Ex quo concludo mothetum *Cum venerint* in principio secundi modi sui sub correctione tanti sui artificis esse defectuosum, ubi tertiam inter duas claves taliter disposuit:

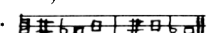


Inter quas notas tonus est una cum semitono maiore et sic

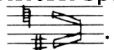
thus they require themselves to minister to the proportions which they serve. For just as we more regularly use – when moving toward the lower part of the dupla proportion – the minor [third], which is less far removed from that part, thus also, when we wish to end in the high part, do we use the sixth which has that same distance from the high part, namely, a minor [third], which [sixth] makes a whole tone above the fifth. And conversely, when we move to the fifth, we widen the third's distance from the lower [part] by using the major third. And thus we measure the same distance, when moving to the fifth, between the octave and sixth. Examples of this may be given in this form:



The third and sixth have therefore become brothers [with respect to] the things that have been mentioned, and also in this [regard], that both have wanted to remain in the superbipartient proportion. As for the distance of the major third, which we said is perfect, we have provided a demonstration above, under b-mi. The distance of the minor third is commonly called [semiditonal] because it consists of a tone and a minor semitone. Yet Boethius more properly calls it tri-hemitonal, because it consists of three semitones, namely two minor ones and one major, which is the interval or distance between a and c, that is, a-re and c-fa-ut. And this is found by cutting a sesquioctava from the sesquitertia proportion. For example, the proportion of 36 to 27 is sesquitertia. In between these numbers I shall take 32, to which the number 36 relates in the proportion of the tone, which is called sesquioctava. If this proportion is removed from the sesquitertia, which constituted the fourth, what shall remain is the proportion you seek, namely that of the third, between 32 and 27, which is called superbipartiens five 27ths.

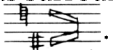
And there are no more thirds than the two which we have mentioned, just as [there are] neither any other sixths than the abovesaid, which I leave to the experience of the hearing. From this I conclude that the motet *Cum venerint*, at the beginning of its second modus under the correction of such artifice, is defective when it disposes the third between two keys in this way: 

These notes are separated by a tone plus major semitone, and thus by

minus quam tertia dytonalis, que perfecta dicitur, plus vero quam semiditonalis, que imperfecta dicitur; nam si littera **b-mi** primam non prederet, esset inter eas precise tonus, ad quam distantiam littera **b-mi** apposita addit semitonium maius per primam suppositionem partis precedentis. Pro excusatione tamen tanti artificis potest dici, quod debeat admitti, quia asperitas eius dulcibus circumstantiis e[s]t suffulta. Sic et secunda admittitur in moteto *Florens vigor* super verbo *Mardocheo* et secunda et quarta in tenore moteti *Rex quem metrorum*. Et septima admittitur, dum ad quintam cantus descensum petierit et ei circumstiterit spatio duorum semitoniorum minorum, ut hic .

In hoc exemplo distat utrumque extremum ipsius septime equaliter ab utroque extremo ipsius quinte, ad quam cantus declinat, spatio semitonii minoris. Hinc est, quod credo huiusmodi septimam minorem claudere duriciem, quam si inequaliter extreme distarent, utputa littera **b-mi** deleta; vix enim ad consonantiarum collegium sexta, que imperfecta est, supradicte septime meruit anteferri. Qualitercumque tamen septima disponatur, nullomodo eam inter consonantiarum genera computabo; et quod admitti[t]ur, facit dulcedo circumdans eam, que amaritudinem eius, ut in aliis dixi, removet et expellit. Septimam ergo veluti secundam et alias proportiones a quatuor supradictis cum Boetio ekmelicas, id est non melicas, merito appellamus.

In sincopationibus insuper videmus sepiissime mediam vel partem maiorem ipsius note in discordantiis disponi, dumtamen prima pars note in concordantia situetur. Impetuosior namque sonus est in principio quam in fine, et principium maxime intenditur in talibus; et hinc est, quod in tempore imperfecto secundam semibreve in dissonantiam ponimus frequentius quam primam, et in tempore perfecto nunc mediam, nunc ultimam, raro tamen utramque, ut maior pars in consonantia disponatur; sic suo modo de perfecto modo et imperfecto,

less than the major third, which is called perfect, but more than the minor, which is called imperfect. For if the letter **b-mi** [♯] did not stand before the first [note], then there would have been precisely a tone between them. When the letter **b-mi** is appended, [however,] it adds a major semitone to this distance according to the first supposition of the preceding part. However, in defense of such artifice it may be said that one should allow it, since its asperity receives support from the sweet things around it. Thus, also, is the second admitted in the motet *Florens vigor* on the word “Mardocheo,” and the second and fourth in the tenor of the motet *Rex quem metrorum*. The seventh is also admitted when it has sought the song’s descent to the fifth, and has surrounded [that fifth] by a space of two minor semitones, as here: .

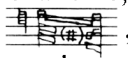
In this example, the extreme ends of the seventh are both equally far removed from the extreme ends of the fifth, towards which the song is directed, [that is,] by the space of a minor semitone. For this reason I think that this seventh contains less hardness than if the extreme ends were at unequal distances [from the fifth], for example if the letter **b-mi** [♯] was erased. For the sixth, which is imperfect, scarcely deserves to be ranked before the abovesaid seventh in the collegium of consonances. Yet however the seventh may be disposed, in no way shall I reckon it among the genera of the consonances. It is only on account of the sweetness surrounding it – which removes and expels its bitterness, as I have said elsewhere – that it is admitted. With Boethius, therefore, we justly call the seventh, like the second and the other proportions from the four abovesaid ones, ekmelic, that is, not melic.

In sincopations, moreover, we see very often that the middle or greater part of this note is disposed in discordance, so long as the first part of the note is situated in concordance. For a sound is more impetuous at the beginning than at the end, and the beginning is greatly strained in such things. It is for this reason that we place the second semibreve in imperfect tempus more frequently on a dissonance than the first [semibreve], and in tempus perfectum sometimes the middle [semibreve], sometimes the last, but rarely both, in order that the greater part be disposed in consonance. The same is true of the perfect modus and the imperfect, provided that

dumtamen mora in dissonantia facta non cum amaritudine sua inficiat auris auditum. Permittit autem auris in modicis se decipere, sed non in immensis, pulsata namque corda tot sonos distinctos reddit, quotiens aer pellitur ex diversis lateribus ipsius corde; inter quorum quoslibet dictat ratio quietam mediam, quam propter eius parvitatem auris nequiens percipere omnes sonos iudicat esse unum, sicut oculus puras partes aeris in ludo, qui turbo vel turbus dicitur, propter velocitatem virge rubre extornate non potens comprehendere occasione spatii brevis temporis totum conum iudicat esse rubrum.

Ex predictis patet plura esse consonantiarum genera quam dissonantiarum; sed quia proportionones consonantiarum in puncto consistunt indivisibili, a quo puncto dum deficimus, in dissonantiam deliramus; hinc est, quod cum maiori difficultate sine magno exercitio consonantias modulamur. —

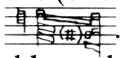
Sexta, que tonum supra dyapenthe amplectitur, perfecta dicitur; cuius proportio in numeris supra in E-la-mi satis est ostensa. Proportio autem sexte alterius, que imperfecta dicitur, potest inter numeros sic investigari: umeri primi semitonii minoris, ut dixi supra, sunt 243 et 256, quorum minimo diviso in tres partes facient eius due tertie numerum 162. Erit autem proportio maximi ad minimum, puta 256 ad 162, in qua imperfecta sexta quiescit, vocaturque superpartiens nonagintaquatuor centesimas sexagesimas secundas.

Inter tertiam et sextam propter vicissitudinem, quam habent adinvicem, moderni similitudinem adauxerunt, quod sicut tres tertias se invicem sequi licet, ita et tres sextas, hoc ideo statuentes, ut cantus ille, qui per tertias et sextas imperfectus censetur, non tamen discors aures trahat et alliciat, ut perfectionem cantus, qui per quintam sequetur vel octavam, quarum tertie et sexte sunt nuntie et ancille, expectatam diutius indicent dulciorem, ut hic , non autem hoc statuentes in quintis vel octavis, ne auris cesset ab advertentia, putans, quod habito fine cesset motus.

the lingering on the dissonance will not infect the hearing of the ear with its bitterness. For the ear may be deceived in moderate things, but not in immense things. For when a string is plucked, it produces as many distinct sounds as often as the air is pushed from the various sides of this string, between any of which [sounds] reason calls for a middle at rest. And since the ear is unable to perceive it because of its smallness, it judges all sounds to be one, just as the eye, in the game which is called top or spintop, judges the pure parts of the air to be red because of the velocity with which the red stem is turned around, as it is unable to take in the whole cone on account of the short time interval.

From the aforesaid things it is apparent that there are more genera of consonances than dissonances. Yet since the proportions of the consonances are situated in an indivisible point, from which point we [immediately] stray into dissonance when we deviate, it is difficult to produce consonances without much training.

The sixth which embraces the tone above the fifth is called perfect. Its proportion in numbers has been shown sufficiently above, under E-la-mi. Yet the proportion of the other sixth which is called imperfect may be investigated in numbers as follows. The shoulders of the first minor semitone, as I have said above, are 243 and 256; if the least of these [numbers] is divided into three parts, then two of its third parts together shall make the number 162. Now the imperfect sixth shall rest in the proportion of the greatest to the least, that is, between 256 and 162, and it is called superpartiens ninety-four 162nds.

The moderns have increased the similitude between the third and sixth because of the way they change relative to each other, so just as it is licit for three thirds to follow each other, so [it is] also for three sixths. They instituted this in order that a song which is reckoned imperfect but not discordant because of the thirds and sixths may draw and attract the ears, so that [the thirds and sixths] may announce the expected perfection of the song which shall follow the fifth or octave (of which the third and sixth are messengers and handmaids), as here: . Yet they did not institute this in fifths or octaves lest the ear would stop being attentive, thinking that all motion ceases when the end is there.

Inspecta ergo tanta fraternitate inter tertiam et sextam quero, numquid due tertie constituent unam sextam. Dic quod non; nam due tertie imperfecte deficiunt a quinta in semitonio maiore; perfecta vero et imperfecta quintam precise constituunt; due vero perfecte quintam superant in semitonio maiore, et sic uno commate excedunt sextam imperfectam, a sexta autem perfecta deficiunt semitonio minore, ut hec omnia ex hiis, que dicta sunt, clare possunt demonstrari.

Hiis visis luculenter arguitur nullas duas proportiones equales, quarum quelibet per se consonantiam redolet, consonantiam posse parere, ut sic ex consonantiis equalibus una consonantia oriatur, nisi sint ad duplam multiplices, ut sunt quadrupla, octupla, sedecupla et sic in infinitum.

Quoniam si quinta quinte coniungitur, oritur una nova proportio: dupla sesquiquarta ut 9 ad 4; si tertia tertie eiusdem quantitatis superponitur, oritur plus vel minus quam quinta vel sexta, ut iam dictum est, cum nulla tertia per medium quintam vel sextam scindat. Ratio autem, quare multiplices ad duplam semper consonant, est propter apparentem terminorum suorum ydemptitatem; quamcumque enim proportionem secum compatitur gravior simplicis duple terminus, eandem patitur et acutus, ut quodammodo duo soni quasi unus individuus totaliter videantur; quo fit, ut dupla cuilibet principali consonantie ad alterum eius extremorum continue possit copulari, ut sub tertia vel supra eam, supra quintam vel sub ea, et eodem modo coniungitur dupla ipsi sexte.

Compatientissima est ergo ipsa dupla, que nulli consonantie dulcedinem minuit, sed potius sui benignitate se prebet gratissimam nec alicui ut sancta mater ecclesia ad se claudit gremium revertenti.

Longe minus compatiens dicitur ipsa quinta, nam seipsam non patitur, quoniam ymo sibimet coniuncta discordiam

Having thus observed so much brotherhood between the third and sixth, I ask if it is possible for two thirds to constitute one sixth. I say that [it is] not. For two imperfect thirds are a major semitone short of the fifth, and the perfect and imperfect [thirds] constitute the fifth precisely, but two perfect ones exceed the fifth by a major semitone, and consequently they exceed the imperfect sixth by one comma, [and] yet are a minor semitone short of the perfect sixth. All these things can be demonstrated clearly from the things we have said.

Having seen all this clearly, we now argue that it is not possible for two equal proportions, each of which by itself has the savor of consonance, to give birth to a [new] consonance, for in this way one consonance would be born from equal consonances — unless, that is, they are multiples of the dupla, like the quadrupla, octupla, sedecupla, and so on, in infinity.

For when a fifth is joined to a fifth, a new proportion is born, the dupla sesquiquarta, 9 to 4. And when a third is superimposed upon a third of the same quantity, [an interval of] more or less than a fifth or sixth is born, as has already been said, since there is no third that splits a fifth or sixth [exactly] through the middle. Yet the reason why multiples are always consonant with the dupla is because of the apparent identity of its boundaries. For whatever the proportion which the lower term of a simple dupla [is able to] tolerate, the same is tolerated by the high one as well, so that two sounds would in some way seem like one totally indivisible [sound]. This is why the dupla of any principal consonance may be coupled to the other of its extreme ends continuously, as under the third or above it, above the fifth or under it. And in the same way the dupla is conjoined with this sixth.

This dupla is therefore the most tolerant. It does not diminish the sweetness of any consonance, but rather, by its kind nature, manifests itself as the most welcoming, and like the holy mother church it does not close [its] bosom to someone who returns.

But the fifth is called less compassionate by far, since it does not tolerate with itself something that is joined to it, but rather is in labor [to bring

parturit et nascitur amaritudo ex duorum dulcium iuxtapositione.

Sic nec tertiam dignatur super se aut ea pedibus calcata superbit, sed eam patitur infra se, quasi eam ventre deglutisset.

Patitur et sextam sub et supra se, non sui compatentia, sed alterius extremorum duple, ad quod extremum sexta coniuncta quinte tertiam sapere iudicatur, ut si gravior sonus sit in clave c-fa-ut, secundus in g-sol-re-ut, tertius in E-la-mi clave acuta, constat c-sol-fa-ut duplam ad c-fa-ut et sic acutam clavem E-la-mi tertiam super duplam.

Et hec eadem causa est, quare seipsam non patitur sub se vel supra se, quia tonus, qui tunc fieret extra limites ipsius duple, sicut tonus, qui fit infra duplam, non pro consonantia computatur.

Est ergo solutio in promptu, quando latitudo seu intervallum duarum vel plurium proportionum sibi invicem coniunctarum duplam excedit proportionem vel duple multiplicem, utrum et quare huiusmodi coniunctio consonet aut dissonare debeat, quia semper habebimus oculum ad proportionem excessus, quo talis coniunctio excedit duplam aut duple multiplicitem.

Gratia huius quero, numquid due triple consonantiam faciant. Videtur, quod sic. Nam prima facie due triple videntur facere unam sextuplam et una sextupla tres duplas.

Sed quotiens dupla duple coniungitur, consonantia generatur, ergo et cetera. Dic, quod non. Quoniam nulla noncupla consonat, ut sunt 9 ad 1; constat enim noncupla ex octupla et ex una sesquioctava, quia proportio 9 ad 1 componitur ex intermediis, puta ex illa, que est 9 ad 8, et ex illa, que est 8 ad 1; sed extremitates proportionis octuple in sonis censentur eedem, ut dictum est supra, eo quod octupla est multiplex ad duplam; et sesquioctava proportio, in qua

forth] a discord, and [in this way] bitterness is born from the juxtaposition of two sweet things.

Neither, therefore, is the fifth considered worthy [to have] the third above itself, nor does it have the pride of trampling it under foot. Yet it does suffer [the third] within itself, as if it had swallowed it in the belly.

It also suffers the sixth both beneath and above itself, not because of its own tolerance, but rather that of either dupla of [its] extreme ends, [in relation] to which [duplas] the sixth adjoined to the fifth is judged to have the savor of a third; as when the lower sound is on the key of c-fa-ut, the second [sound] on g-sol-re-ut, and the third on the high key of E-la-mi. Then c-sol-fa-ut is the dupla of c-fa-ut, and E-la-mi, consequently, makes the high key [of] the third above the dupla.

And this is also the reason why [the fifth] does not suffer *itself* beneath or above itself, for the tone which would then be outside the limits of this dupla [that is, d-c] is not considered a consonance, nor the tone which would be within the dupla [that is, f-g].

When, therefore, the width or interval between two or more proportions that are joined together exceeds the dupla proportion, or a multiple of the dupla, then this readily explains if and why this kind of conjunction is consonant or dissonant. For we shall always focus [our] eye on the proportion of the surplus by which the conjunction exceeds the dupla or a multiple of the dupla.

For this reason I ask whether it is possible for two triplas [i.e., twelfths] to make a consonance. And it would seem that it is. For at first sight two triplas seem to make one sextupla, and one sextupla two duplas. [This is erroneous:  $\frac{3}{1} \times \frac{3}{1} = \frac{9}{1}$  and  $\frac{6}{1} = \sqrt[6]{\frac{6}{1}} \times \sqrt[6]{\frac{6}{1}}$ .]

And no matter how many times we may join a dupla to a dupla, it always generates a consonance. Therefore, and so on. Yet I say that [it is] not. For no noncupla, as 9 to 1, is [ever] consonant. For the noncupla consists of an octupla and one sesquioctava [ $\frac{8}{1} \times \frac{9}{8} = \frac{9}{1}$ ], because the proportion of 9 to 1 is composed from intermediate [proportions], namely 9 to 8 and 8 to 1. Yet the extreme ends of the proportion of the octupla are judged to sound the same, as said above, because the octupla is a multiple of the dupla [ $\frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} = \frac{8}{1}$ ]. And the sesquioctava proportion, in



tonus quiescit, non causat consonantiam sub vel supra duplam, cum non sint nisi quatuor consonantie principales, ergo nec supra octuplam.

Sed due triple cons[t]ituunt noncuplam, nam proportio 9 ad 1 componitur ex duabus proportionibus inter hos tres numeros 9. 3. 1., inter quos due sunt triple, et est maximi ad minimum noncupla, ergo et cetera.

Nec valet argumentum in oppositum nisi contra huius scientie penitus ignaros, quoniam due triple constituunt ultra sextuplam unam integram sesquialteram, ut est inter hos tres numeros 9. 6. 1. leviter manifestum; est enim medii ad minimum sextupla, et maximi ad medium sesquialtera, sed proportio maximi ad minimum continet duas triplas, ergo et cetera.

Item inter hos quatuor numeros 8. 4. 2. 1. sunt tres duple, et est maximi ad minimum octupla, que in sesquitercia proportione vincit sextuplam, ut hic 8. 6. 1.; sic ergo non faciunt duas triplas tres duple, sed sesquioctavam minus.

Sed an sextupla consonet? Dic, quod sic, quia facit sesquialteram, id est quintam vel dyapenthe, supra quadruplam, que quadrupla multiplicat ipsam duplam et per consequens censebitur eadem.

Sed quid de quintupla? Dic, quod dissonat, quia sesquiquartam saporit ut 5 ad 4, quam inter manuales claves impossibile est disponi; est enim maior quam dytonus supra quadruplam ut hic 338. 324. 256., minor vero quam dyatesseron ut hic 15. 16. 12.

Unde ad hoc, quod aliqua proportio sonorum sit inter claves signabiles, requiritur, quod uterque numerus primus eius sit harmonicus, id est divisibilis continue ad unitatem per numerum ternarium, ut 1. 3. 9. 27. et cetera, et talis est masculinus; vel per binarium, ut 1. 2. 4. 8. et cetera, et talis est femininus; vel viceversa, ut 6. 12. 18. 24., et talis est promiscuus, quia generatur ex ductu masculini et feminini.

which the tone resides, does not make a consonance either under or above the dupla – since there are only four principal consonances – and therefore not above the octupla either.

Yet two triplas constitute the noncupla, for the proportion 9 to 1 is put together from two proportions between these three numbers: 9. 3. 1. Between these there are three triplas, and the greatest [number] makes a noncupla relative to the smallest, therefore, et cetera.

Nor does the argument to the contrary have validity except to those who are completely ignorant of this science. For two triplas are a whole sesquialtera in excess of the sextupla, as is easily apparent in these three numbers: 9. 6. 1. For there is a sextupla between the middle [number] and the smallest, and a sesquialtera between the greatest and the middle. But the proportion between the greatest and smallest includes two triplas, therefore, et cetera.

Also, between these four numbers: 8. 4. 2. 1. there are three duplas, that is, an octupla of greatest to smallest, which is greater than the sextupla by a sesquitercia proportion, as here 8. 6. 1. Therefore, two duplas do not make two triplas, [except if we] subtract a sesquioctave [from them].

But would the sextupla make consonant sound? I say that it does. For it makes a sesquialtera, that is, a fifth or diapente, above the quadrupla. And the quadrupla multiplies the dupla, and shall therefore be deemed the same [as a dupla].

But what about the quintupla? I say that it is dissonant. For the sesquiquarta means 5 to 4, and it is impossible to dispose [this proportion] among the keys of the hand, as it is greater than the major third above the quadrupla, as here: 338. 324. 256, but smaller than the fourth, as here: 15. 16. 12.

Therefore, in order that some proportion of sounds may be [included] among the signable keys, it is necessary that every first number be harmonic, that is, continuously divisible by a ternary number down to unity, like 1. 3. 9. 27, and so on, and this is masculine. Or by a binary [number], like 1. 2. 4. 8, and so on, and this is feminine. Or the other way round, like 6. 12. 18. 24., and this is mixed, since it is generated from the masculine and feminine series.

Hec descriptio numeri armonici trahitur ex libro Navarrine universe et potest in tabula pulchra depingi, ut hic:

1.	3.	9.	27.	81.	243.
2.	6.	18.	54.	162.	486.
4.	12.	36.	108.	324.	972.
8.	24.	72.	216.	648.	1944.
16.	48.	144.	432.	1296.	3888.
32.	96.	288.	864.	2592.	7776.

et sic in infinitum procedendo. In cuius figure lateribus invenies cuiuslibet proportionis musicalis numeros primos. Unde tertius masculinus et quartus femininus faciunt tonum, sic et sextus masculinus et octavus femininus faciunt semitonium, item octavus masculinus et duodecimus femininus faciunt apothomen, et cetera; sic suo modo de promiscuis. Ex quibus patet quod 5, 7, 11 et cetera non sunt numeri armonici, ergo nec proportiones, quas ipsi cum aliis quibuscumque constituunt, sunt signabiles inter claves.

Habito autem oculo ad proportiones quatuor consonantiarum principalium et ad proportionem duplam et que duplam multiplicat, nullo modo in proportionibus, que latitudinem duple proportionis superant, an consonantiam pariant nec ne, quis decipietur.

Iusta ergo de causa veteres moti fuerunt claves duple proportionis eadem littera consignare. Huius etiam duple intuitu tanta inter sextam et tertiam nata est amicitia, quod quasi dilectione sororia altera alterius gaudet lavare pedes. Et forte si natura sextam peperisset flexibilem, recompensaret tertie brachiorum mutuos et vicissitudinarios amplexus.

This description of the harmonic number is drawn generally from the Navarrese book, and it can be depicted in a beautiful table, as here:

1.	3.	9.	27.	81.	243.
2.	6.	18.	54.	162.	486.
4.	12.	36.	108.	324.	972.
8.	24.	72.	216.	648.	1944.
16.	48.	144.	432.	1296.	3888.
32.	96.	288.	864.	2592.	7776.

and proceeding thus into infinity. On the sides of this figure you will find the first numbers of any musical proportion. So the third masculine [number, which is 9] and the fourth feminine [8] make a tone. Likewise also the sixth masculine [243] and the [ninth] feminine [256] make a semitone. And the eighth masculine [2187] and twelfth feminine [2048] make the apothome, and so on. And thus also the neutral ones in their own way. From this it is apparent that 5, 7, 11 and so on, are not harmonic numbers, and therefore neither are the proportions which they make with any others that are signable among the keys.


Once one has seen the proportions of the four principal consonances, and also the dupla proportion, and the [proportions] which multiply the dupla, no one shall be mistaken in any way about the proportions which exceed the side of the dupla proportion, whether they produce consonant sounds or not.

It was therefore with good reason that the old decided to mark the keys of the dupla proportions with identical letters. Out of regard of the dupla, such friendship was born between the sixth and third that each rejoices in washing the feet of the other, as if out of sisterly love. And if nature brought forth a flexible sixth, then perhaps it was to repay the mutual and changing embraces of the arms of the third.

Sed est fortior ratio signanda in coniunctionibus earum, que simul capte ad latitudinem nequaquam perveniunt ipsius dyapason, ut est duarum tertiarum equalium et ipsius tertie sub vel supra quintam. Cuiusmodi rationem natura nobis occultans sicut infinita alia sibi specialiter reservavit.

Et si dicas, quod ratio sufficiens capitur ex habitudine sonorum adinvicem in debita constantium proportione, quero, quare hec proportio concordat et non illa; et sic finaliter fiet recursus ad ipsam naturam, que a priori cuncta noverat. Nos autem ea modica, que scimus, a posteriore divinamus.

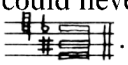
Tria sunt genera consonantiarum per accidens: Tonus cum duobus semitoniis minoribus, id est dytonus solo commate imperfectus, tritonus et dyatesseron. Que merito consonantie per accidens vocantur, nam per se sine adiunctione supradictarum consonantiarum aures stupefacerent potius quam mulcerent. Sonant autem supradicte, que principales sunt, absque aut una cum istis; istas vero sine illis impossibile est esse. Possibile est enim corpus luminosum esse sine umbra, hanc tamen impossibile est esse sine illo. Has itaque consonantias umbre ut umbrosas, illas vero lucido corpori ut luminosas merito comparamus.

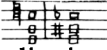
Numquam enim tonus cum duobus semitoniis minoribus, id est dytonus cui comma deficit, consonantiam faceret, nisi super tertiam perfectam poneretur ut hic . Inter quarum duas extremas sexta est, que, licet debilis sit, admittitur et, ut dixi supra, pro una de principalibus consonantiis computatur; est et tertia inter mediam et graviorem propter appositionem littere b-mi ipsam mediam precedentis; inter quam mediam et acutam a dytonali latitudine deficit solum comma, eo quod reiectis litteris b-mi et b-fa tritonus integer subsisteret, qui occasione dictarum litterarum nunc eclipsim patitur per duo semitonia maiora; perdit ergo tritonus integrum tonum et comma; remanet ergo tonus cum duobus semitoniis minoribus, id est dytonus perfectus solo commate mutilatus.

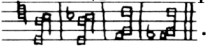
Yet the reason for marking is stronger in combinations of those [thirds and sixths] which, when added up, do not reach the side of the octave at all, such as [combinations] of two equal thirds, or of the third beneath or above the fifth. Nature hides this reason from us, just as it has specially reserved for itself infinitely many other things.

And if you say that the relationship of the sounds to each other, in the requisite proportion of constant things, provides sufficient reason, then I ask why this proportion is concordant and that one not. And so, ultimately, there must be recourse to nature, which has known all things beforehand. Yet the few things that we know we only guess afterwards.

There are three genera of consonances in an accidental sense: the tone plus two minor semitones, that is, the major third which is imperfect only by a comma, the tritone, and the fourth. These are deservedly called consonances in an accidental sense, for by themselves alone, without the addition of the abovesaid consonances, they sooner stupefy the ears than soothe them. For the abovesaid principal [consonances] sound without or with the latter. But it is impossible for the latter to be without them. It is possible for a light-giving body to be without a shadow, but impossible for the latter to be without the former. Therefore we justly compare the [accidental] consonances to the shadow, [being] shadowy, but the [principal consonances] to the shining body, [being] light-giving.

For the tone plus two minor semitones, that is, a major third lacking a comma, could never make a consonance unless placed above a perfect third as here: . There is a sixth between the two extreme ends of [these thirds] which is admissible even though it is weak, and reckoned among the principal consonances, as I said above. There is also a third between the middle and lower [notes], because the letter b-mi [♯] is placed before the middle [note]. Between the middle and high [notes] there is a major third which lacks only a comma. For if the letters b-mi [♯] and b-fa [♭] were removed [from the example], there would be a whole tritone left [between f and b♭]. But because of the said letters, [this tritone] now suffers an eclipse by the two major semitones. Consequently the tritone loses a whole tone plus comma, and therefore only a tone plus two minor semitones remains, that is, a perfect major third mutilated by a single comma.

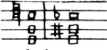
Tritonus, si per se proferatur, aures sua duricie terribiliter effugabit; quapropter veteres ipsum tritonum odientes tetracordum coniunctum in nostro dyatonico cantu — ideo sic dicto dyatonico, quia per duos tonos addito minori semitonio progreditur — per dyatesseron ordinave>runt. Eius tamen duricie non obstante admittitur per accidens super tertiam imperfectam, ut hic . In predictis duobus exemplis sunt omnes tres soni adinvicem compassibiles, quorum si gravior sonus desit, residuorum sonorum consonitus peribit tanquam diruto fundamento.

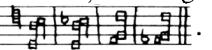
Secundam non experior consonare, nec per se, nec cum aliis, quod tamen multis visum est de tono vel minore semitonio addito dyapenthe, ex quo quelibet singulariter per se tam dyapenthe quam sexta consonat; et ea propter ipsam secundam non audeo, quamvis alias oppositum cogitaverim, inter septem consonantias aliququaliter numerare, ut hic .

Mirabiles ergo sunt effectus nature etiam in sonis, quod magis dulce dulcedinem amaricat ac minus dulce hoc, quod insipidum et amarum est, sapidum reddit et dulcorat, ut hec in dictis speculari potes exemplis.

Sexta namque per se dulcis est, quam maior dulcedo, ut est ipsius dyapenthe, velut maius lumen ipsum minus suffocat, si eidem non per appositionem, puta ad ipsius quinte pedes aut humeros. sed per uterinam conceptionem quodammodo misceatur, ut in ultimo exemplo supradicto. Minus tamen dulcis est tertia quam quinta, et in infinitum durior tritonus quam sexta, hanc nichilominus tritoni duriciem ipsa modica tertie mollicies, dum tamen per pedem stringit, ut David Goliam superat, sed reminiscens sororie dilectionis hunc moribundum tritonum oleo dulcedinis sue misericorditer inungit.

Cuius effectus si omnimodam rationem conaris ostendere per hoc, quod acutior pars tritoni cum graviori parte huius tertie residet in proportione congrua, que sexta dicitur, cur non eandem ostendis in sexta, que quintam concipit, ut in exemplo

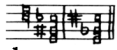
If the tritone is performed by itself, it shall horribly frighten the ears with its hardness. For this reason the old, who hated this tritone, arranged the conjunct tetrachords in our diatonic song by fourths — which is why it is called diatonic, for it proceeds by two tones with the addition of a minor semitone. Notwithstanding its hardness, however, [the tritone] is admitted on top of an imperfect third in an accidental sense, as here: . In the aforesaid two examples all three sounds are compatible with each other. But if of the lower sound of these is lacking, then the noise produced by the remaining sounds shall drash down as if the foundation [were] demolished.

I do not perceive the second as consonant-sounding, neither by itself nor with others, although it has seemed [consonant] to many when the tone or minor semitone is added to a fifth. Each [of these], fifth as well as sixth, sounds consonant singularly by itself; but because of the second I am not prepared to reckon them among the seven consonances, although I would have considered the opposite elsewhere, as here: .


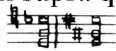
The effects of nature in sounds are wondrous, therefore, in that it makes sweetness bitter more sweetly, and renders tasteful and sweetens, less sweetly, that which is insipid and bitter, as you can examine in the said examples.

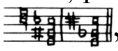
For the sixth is sweet by itself. But the greater sweetness of the fifth, [which is] like a brighter light, does not choke it so much when it is blended with the same, not by apposition — for example, at the feet or shoulders of the fifth — but in some way by uterine conception, as in the abovesaid last example. The third, however, is less sweet than the fifth, and the tritone infinitely harder than the sixth. Yet the moderate softness of the third nevertheless ties up the hardness of the tritone at the feet, like David defeating Goliath, though it mercifully anoints this moribund tritone with the oil of its sweetness, being mindful of sisterly love.

If you try to show every sort of reason for this effect, [namely,] that the higher part of the tritone makes an agreeable proportion with the lower part of this third, which [proportion] is called the sixth, why do you not show the same [reason] for the sixth which conceives the fifth,


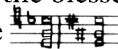
predicto. Cur etiam ratio tua claudicat in duabus sextis inequalibus et in tertiis, que adinvicem colluctantes, imperfectioribus forte prostratis et occumbentibus, stallos in choro et voces in capitulo insimul perdiderunt, ut hic  ubi due sexte seinvicem, si simul proferantur, sicut et due inequales tertie mirabiliter perturbant. Et tamen utrarumque termini acuti sua fundamenta respiciunt in debita proportionum habitudine, prout superius est ostensum.

Causam ergo ignorantes effectum sensibiliter percipimus, quod dum acutiores termini quarumcumque duarum consonantiarum in debita proportione super unum et idem fundamentum fundati seinvicem solo tono vel minori spatio excedunt, tales due consonantie ut duo vicini proximi, inter quos lis finium regendorum, — meta est spreta — compositione quacumque sese litigando depauperant et metu sinistre contrarie tandem seinvicem miserabiliter occidunt.

Dyatesseron, quam quartam dicimus, ex qua secundum Boetium consonat, inter has per accidens, secundarias et proprie accessorias consonantias merito collocatur; per se namque nulla est. Sed ut dyapason inter principales, ita hec inter secundarias dulcissime vociferat, dum pro reciprocis basiis ora dulciter coniungentes dyapason et dyatesseron invicem se salutant, ut hic: . Tantum namque vite spiraculum dupla per huiusmodi osculum ipsi dyatesseron inspirare videtur, quod ab ipsa et in ipsam natura cantum incipi precipit et finiri, quale privilegium natura aliis consonantiis omnibus, etiam si mille existerent, dupla tamen et quinta exceptis, concedere dedignatur. Nec mirum, si in hoc casu ipsam dyatesseron fides sua salvam facit, quia fundata est supra firmam petram, dum ad ipsius [ipsius] procidere dyapenthe dignata est devota pedum oscula beatorum. Dyatesseron supra quamvis tertiam consonantiam redolet, ut hic , ubi tertia caritativo sororis intuitu colla iugo subdere ipsius dyatesseron exultat, ac eidem suas mellifluas sicut tritono solet dividere facultates.

as in the aforesaid example? Why also is your reason limp in [the case of] two unequal sixths and thirds that have ruined the stalls in the choir and the voices in the chapter, because they struggle with each other, perhaps struck down and killed by more imperfect ones, as here: , where two sixths, if performed at the same time, disturb each other in an astonishing way, just as do two unequal thirds. And yet the higher boundaries of both respect their foundations in the requisite proportional relationship, as shown above.

Therefore we perceive the effect with the sense, but do not know the cause why, when the higher boundaries of any two consonances that are founded, in correct proportion, upon one and the same foundation, exceed each other by only a tone or a lesser interval, those two consonances [are] like two neighbors involved in a boundary dispute, who end up impoverishing each other by lawsuits and, having spurned every settlement, miserably kill each other in the end in fear of an adverse contrary [judgement].

The diatessaron, which we call fourth, which according to Boethius makes a consonant sound, is deservedly placed together among the secondary and properly accessory consonances; for in itself it is not [a consonance] at all. But just like the octave among the principal ones, so this one clamors most sweetly among the secondary ones, [namely,] when the octave and fourth greet each other, sweetly joining [their] mouths in reciprocal kisses, as here: . For by this kiss the dupla seems to breathe so great a breath of life into this fourth that nature teaches [us] to begin and end the song with this [octave]. Nature has refused to grant this privilege to all other consonances, even if there were thousands, excepting however the dupla and fifth. Nor is it to be wondered if, in this case, its faith brings salvation to the fourth, for it is founded upon a firm rock, while the fifth has been deemed worthy to fall prostrate before it, [rendering] devout kisses on the feet of the blessed. The fourth above any third is redolent of consonance, as here , where the third, out of the charitable regard of a sister, rejoices to place [its] neck under the yoke of the fourth, and is accustomed to share with the same [fourth] its mellifluous faculties, just as [it does] with the tritone.

Dyatesseron supra seipsam nequaquam duplam procreat, sed ab ea deficit in sesquioctava proportione, ut supra in f-fa-ut demonstravi, et sextam exsuperat: ergo duplata non consonat; sed supra tertiam aut quintam principatum suum retinet; que due ipsi serviunt propter causam supradictam.

De dyatesseron autem, et non de aliis, tamquam meliori consonantia post duplam et quintam veteres tractaverunt duplici fortassis de causa: prima causa dicta est, nam in cantu trium sonorum adinvicem, quo consonantie per accidens discernuntur, longe lascivior magisque iocunda est dyatesseron supra quintam, quam sint alique alie consonantie ubicumque situate post dyapenthe tamen vel duplam. Non etiam dyatesseron alicui sicut tertia famulatur vel sexta; consonantibus namque quinta et dupla, necessario consonabit et dyatesseron; et ideo de hiis, que servilis conditionis sunt non curantes, dumtaxat dominantibus intendetur.

Secunda forte ratio stat in regione vel tempore. Diverse namque regiones diversos cantus exigunt, ut in hoc experimento dum scolas Oxonienses in Anglia colui, quam regionem a Comitatu Hollandie, loco mee nativitatis, solum mare discriminat audito, quod layci ibidem et clerici, senes, iuvenes et indifferenter omnes tertiis et sextis tantam atribuebant affectionem quodque, duplis et quintis postpositis, ipsas solas invocantes quasi adorare videbam; vehementer attonitus de tam vicine regionis diversa natura continue ammirabar.

Tempus huius dubii nexum potest dissolvere; verisimile enim non est, si tempore Pytagore vel Boetii consonuisse tertia et sexta, quoniam de eis ut de dyatesseron fecissent utique mentionem; nec estimandum est tantos viros speculativam numerorum sensuali experientie pretulisse.

Item in questione illa generalis, utrum dyatesseron supra dyapason consonaret, respondent Pythagorici, quod non, eo quod sesquitertia ut 8 ad 6 supra duplam ut 6 ad 3 superpartiens genus constituunt ut inter 8 et 3, abhorrentes ullam

A fourth on top of itself does not produce a dupla by any means, but falls short of it by a sesquioctava proportion, as I have demonstrated above under f-fa-ut, and it exceeds the sixth. Therefore [the fourth] does not sound consonantly when doubled; yet it retains its rule above the third or fifth, both of which two are servants for the abovesaid reason.

Yet the old have treated the fourth, and not the others, as if [it were among] the better consonances after the dupla and fifth, perhaps for two reasons. The first reason has already been mentioned, for in a song of three sounds [sounding] together, in which we discern accidental consonances, the fourth above the fifth is more delicious and more pleasing by far than other consonances that are placed anywhere after a fifth or dupla. But the fourth is not a servant of some other [interval], as the third or sixth [are]. For when the the fifth and dupla are sounding consonantly, then the fourth shall necessarily sound consonant as well. To this extent, therefore, it may be extended from those dominant ones who are not burdened by the servile condition.

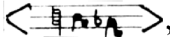
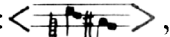
The second reason perhaps has to do with region and time. For different regions need different songs, as I have heard in this experience — when I lived at the Oxonian schools in England, which region is separated from the County of Holland, the place of my birth, only by the sea — that laymen there, and clerics, old men, youths, and indifferently all, felt such affection for thirds and sixths, that, invoking these alone, [and] disregarding duplas and fifths, they seemed to me almost to worship them. Vehemently struck, I have continuously wondered at the different nature of so near a region.

Time may dissolve the constraint of this doubt. Yet it is not likely that the third and sixth sounded consonant at the time of Pythagoras or Boethius, for they certainly did not mention them, like [they did] the fourth. Nor is it to be supposed that so many men would have preferred the speculative [proportion] of numbers above sensual experience.

As for the general question whether the fourth above the octave sounds consonant, the Pythagoreans responded that it [does] not, because they established the superpartiens genus as between 8 and 3 — the sesquitertia, such as 8 to 6, [being] on top of the dupla, such as 6 to 3. And since it was

consonantiam fore in illo genere; ergo nec tertia nec sexta consonant secundum ipsos, ex quo quolibet, ut dictum est, sit in genere superpartienti. Et cum idem sonus gravis ipsius dyapason et acutus censentur, sequitur ex Pithagoricorum opinione, quod nec valebit dyatesseron supra graviorem sonum ipsius duple et per consequens nec sub quinta. Vocabant tamen Pythagorici eam consonantiam, quia, ut dictum est, solum per accidens resonabat.

Ptholomeus tamen eam consonare dicit supra dyapason et excipit dumtaxat primam speciem generis superpartientis, quam dyatesseron cum dupla constituit, et ergo per eum consonat sub dyapenthe per rationem predictam. Sic ergo dyatesseron tempore Ptholomei consonantiam per se fecit.

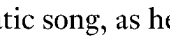
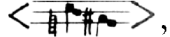
Tertia vero aut sexta per se nec Pytagore nec Ptolomei temporibus umquam consonuit, saltem in regione ipsorum, nostre autem regioni, qua tertiis et sextis, et nullomodo sesquitertie proportioni per se, insistimus. Quam etiam in extraneis vocum varietatibus iuvenes potius quam antiqui lascivire videntur nunc saturati tertia sextam sitiendo, nunc relicta decima in quintam se precipitando, nunc longarum sprete protractione fracturis minutissimis minutiores applicando, perfectum in imperfectum convertendo, nunc sincopis fastiditi ad singultus properando, quos oketus dicimus, ymmo pluries super dyatonico nauseantes non solum chromatico cantui applaudendo ut hic: , sed et novo generi cantus, quod “commaticum” dici potest, inherendo ut hic: , ne dum breviter, ut apparet in sonis, verum in aliis infinitis rebus videtur utique tempus diversitatem maximam influxisse. Absit tamen, quod opinio pravitatis heretice, per quam dyatesseron sicut et cuncta preterita in illa magna celi revolutione reditura sint, obtineat veritatem.

Quatuor atque decem cubis / ter C deme septem.

Dena dies operis.

abhorrent to them that any consonance should be in this genus, they deemed neither the third nor the sixth to be a consonance, as both are in the superpartiens genus, as already said. And since the low and high sounds of the octave are judged the same, it follows from the Pythagorean opinion that the fourth above the lower sound of the dupla shall not be valid, and consequently neither below the fifth. For the Pythagoreans called it a consonance because, as already said, it resounded only in an accidental sense.

Ptolemy however said that it does make consonant sound above the octave, and to this extent he excepts the first species of the superpartiens genus, which the fourth makes with the dupla, and therefore it makes consonant sound under a fifth because of the aforesaid reason. In this way, then, the fourth by itself made a consonance in the time of Ptolemy.

The third by itself, however, or the sixth, has never sounded consonantly in the times of either Pythagoras or Ptolemy, at least in their region, [but it has] in our region, where we insist on thirds and sixths but in no way on the sesquitertia proportion by itself. And youths seem to indulge more than the ancients in strange varieties of sounds, sometimes thirsting for the sixth, being sated by the third, sometimes throwing themselves into the fifth, having relinquished the tenth, sometimes applying more minute fractions than even the most minute ones, having spurned the protraction of longas, converting the perfect into imperfect, sometimes hastening to [sing] the hiccups which we call hockets, having become averse to syncopations, and being sick of diatonic [song], they applaud not just chromatic song, as here: , but also cling to the new genus of song which may be called “commatic,” as here: , much more, as it appears in sounds, does it seem for a moment that time truly has poured the greatest variety into other infinitely many things. Yet God forbid that the opinion of the heretical depravity, according to which the fourth and all bygone things are to be returned to this great revolving of the heavens, shall prevail over the truth.

From four and ten cubes [plus] three hundred, take away seven.

The tenth day of the work.